

Comparing Booster Designs  
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Most Booster designs start with a desired extraction intensity at a desired repetition rate. The beam power delivered is limited to the beam power lost in the tunnel. The amount of joules deposited into the accelerator enclosure per beam pulse due to beam loss is inversely proportional to the repetition rate so that the average beam power lost in the tunnel is a constant:

$$P_L = J_L R \quad (1)$$

Where  $J_L$  is the Joules lost per pulse and  $R$  is the average repetition rate. For the month of March 2005, the Booster ran at  $P_L=440W$ .

Summary for Event 10	
From 01-MAR-2005 00:00:00	
to 01-APR-2005 00:00:00	
Percentage up time:	88.1
Total Events:	13605200
Total Protons:	4.32E+19
Average Events/second:	5.46
Average protons/Event:	3.35E+12
Average protons/hour:	6.58E+16
Maximum protons/hour:	8.33E+16
(protons out)/(protons in):	.828
(Joules lost)/(1e12 prot):	23.7

Figure 1 . Booster Performance for the month of March 2005

For simplicity the beam loss can be divided into two categories, beam loss due to creating the notch for the kicker and beam loss transversely during acceleration.

$$J_L = \alpha_n \Delta N_n + \alpha_A \Delta N_A \quad (2)$$

Where  $\Delta N_n$  is the amount of beam lost during notching and  $\Delta N_A$  is the beam loss during acceleration. The coefficients  $\alpha_n$  and  $\alpha_A$  relate the amount Joules lost to the amount of beam loss for notching and acceleration respectively. These coefficients are a function of the energy in the cycle and are empirically determined from present Booster performance as  $\alpha_n = 71J/1 \times 10^{12}$ proton and  $\alpha_A = 142J/1 \times 10^{12}$ proton. (See Figure 2) The ratio of the amount of beam loss for notching to the injection intensity is given as:

$$\Delta N_n = f_n N_{inj} \quad (3)$$

The ratio of the amount of beam loss for acceleration to the injection intensity is given as:

$$\Delta N_A = f_A N_{inj} \quad (4)$$

The total efficiency is given as:

$$\frac{N_{ext}}{N_{inj}} = (1 - f_n - f_A) \quad (5)$$

If the notching fraction  $f_n$  is given as an input to the design, then the fraction of beam loss during acceleration that can be tolerated is:

$$f_A = \frac{P_L - (N_{\text{ext}} \alpha_n R + P_L) f_n}{N_{\text{ext}} \alpha_A R + P_L} \quad (6)$$

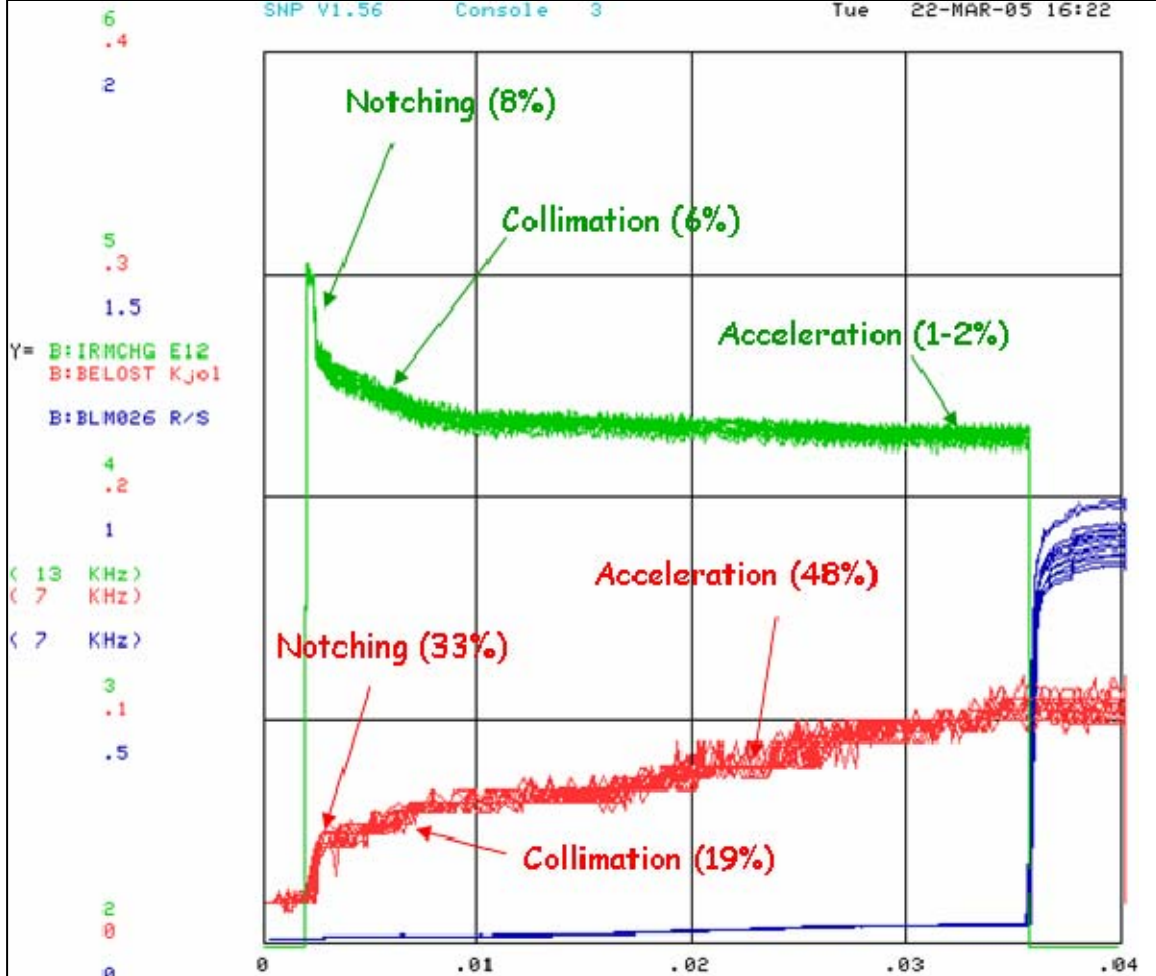


Figure 2. Typical Booster Intensity profile

The next step is to determine the magnet aperture necessary for the required acceleration efficiency. For a Gaussian profile in transverse phase space, the amount of beam in the halo that is outside the aperture is:

$$f_h = e^{-3 \frac{A}{\epsilon_{95}}} \quad (7)$$

where  $A$  is the aperture and  $\epsilon_{95}$  is the 95% emittance. The amount of beam that is permitted to be in the halo is:

$$f_h = \frac{\Delta N_A}{2(N_{\text{ext}} + \Delta N_A)} = \frac{f_A}{2(1 - f_n)} \quad (8)$$

where the factor of 2 comes from the halo in both planes. The aperture required is:

$$A = \frac{S_f \varepsilon_{95}}{3} \ln \left( \frac{2(1-f_n)}{f_A} \right) \quad (9)$$

where an extra “safety” factor,  $S_f$ , was added. This safety factor would have to be about 1.65 to satisfy the requirements in the Proton Driver Study II – Part 1 design report.

We will assume that the minimum beam emittance at injection is determined by the incoherent space charge tune shift:

$$\varepsilon_n = B \frac{3r_o}{2\pi} \frac{N_{inj}}{\beta \gamma^2 \Delta v} \quad (10)$$

where  $N_{inj}$  is the injection intensity,  $\varepsilon_n$  is the normalized emittance at injection,  $\beta$  is the ratio of the velocity of the beam to the velocity of light,  $\gamma$  is the ratio of the beam energy to the rest energy,  $\Delta v$  is the incoherent space charge tune shift,  $B$  is the bunching factor and  $r_o$  is the classical radius of the proton ( $1.53 \times 10^{-18}$  meters).

In the Proton Driver Study II – Part 1 design report, the half aperture of the magnets must exceed:

$$\Delta x = \sqrt{\frac{A_n}{\beta \gamma}} \beta_{max} + \frac{\Delta p}{p} D_{max} + \text{c.o.d.} \quad (11)$$

where  $A_n$  is the normalized acceptance,  $\beta_{max}$  is the maximum lattice beta function,  $D_{max}$  is the maximum lattice dispersion function,  $p$  is the beam momentum,  $\Delta p$  is the momentum spread, and c.o.d. is the closed orbit distortion.

Parameter	Present	Stage 1-2	Stage 3	Stage 4	PD2	
Extraction Intensity	3.4	4.7	4.0	8.0	25.0	$\times 10^{12}$
Rep. Rate	5.46	8	15	15	15	Hz
Average Beam Power Lost	443	443	443	443	443	Watts
Notch Joule Coef	71	35	0	0	0	Joules/ $10^{12}$
Acceleration Joule Coef	143	143	143	143	143	Joules/ $10^{12}$
Notch loss	8.0	8.0	0.0	0.0	8.0	%
Acceleration loss	9.9	5.2	4.9	2.5	0.8	%
Efficiency	82.1	86.8	95.1	97.5	91.2	%
Injection Intensity	4.1	5.4	4.2	8.2	27.4	$\times 10^{12}$
Injection Energy	400	400	400	400	600	MeV
$\beta$	0.71	0.71	0.71	0.71	0.79	
$\gamma$	1.43	1.43	1.43	1.43	1.64	
Allowed Tune Shift	0.47	0.47	0.47	0.47	0.47	
Bunching Factor	2.00	2.00	2.00	2.00	2.00	
Norm. Emittance at Inj	8.7	11.6	9.0	17.6	40.0	$\pi$ -mm-mrad
Gaussian Form Factor	1.625	1.625	1.625	1.625	1.625	
Norm Acceptance at Inj	13.8	22.4	18.1	41.7	119.0	$\pi$ -mm-mrad
F magnet $\beta_x$	33	33	33	15	15	m
F magnet $\beta_y$	14	14	14	20	20	m
F magnet $D_x$	3	3	3	2.5	2.5	m
D magnet $\beta_x$	14	14	14	15	15	m
D magnet $\beta_y$	22	22	22	20	20	m
D magnet $D_x$	2.5	2.5	2.5	2.5	2.5	m
Momentum Acceptance	0.2	0.2	0.2	1.2	2.4	%
Closed Orbit Tolerance	13	6	10	20	20	mm
F Aperture Width	2.42	2.60	2.54	3.92	6.07	in
F Aperture Height	1.60	1.62	1.64	3.04	4.16	in
D Aperture Width	1.79	1.82	1.83	3.92	6.07	in
D Aperture Height	1.87	1.97	1.95	3.04	4.16	in

Figure 3. Example Spreadsheet. The parameters in grey are outputs of the model.